Understanding the Far-Field Properties of Orbital Angular Momentum Beams through the Antenna Aperture Field Method

Anastasios Papathanasopoulos, Junbo Wang and Yahya Rahmat-Samii
Electrical and Computer Engineering Department
University of California Los Angeles
Los Angeles, CA, USA
anastasispapath@g.ucla.edu, wangjunbo@ucla.edu, rahmat@ee.ucla.edu

***Abstract*—This work aims to provide insight into the far-field properties of orbital angular momentum (OAM) beams. Semi-analytical expressions for the far field of a general cylindrically-symmetric OAM aperture field are derived using the antenna aperture field method. The far-field properties of a circularly-polarized OAM carrying fields are considered. In particular, we derive analytical expression for the minimum theoretically achievable AR of a circularly-polarized OAM beam. For a common OAM-carrier aperture field distribution, i.e., the Laguerre-Gaussian distribution, analytical expressions of the fields are obtained in closed forms. The equations obtained in this paper show excellent agreement with predictions obtained from full-wave simulations. The derived analytical and semi-analytical formulas can potentially guide the design of OAM communication systems.**

I. INTRODUCTION

Driven by the recent increasing scientific interest and research on OAM communications, we revisit the fundamentals of OAM beams and provide insight into their far-field properties [1]. Our main analysis tool is the antenna aperture field method, which is well-established in electromagnetics textbooks and widely applied in the analysis of conventional antennas [2]. To the authors best knowledge, there has not been a comprehensive and systematic characterization of the far-field properties of OAM beams. The significance of understanding the far-field properties of OAM beams is manifested as the following: (i) in many application scenarios, an OAM-carrying radiation is characterized in the far-field. Therefore, the understanding of OAM's far-field wavefront, polarization, phase, etc. can be crucial to establishing effective OAM communications links and understanding inherent limitations; (ii) analytical or semi-analytical expressions for the OAM far-fields can provide proper guidelines to the design of OAM antennas (an example can be found in the domain of conventional reflector antennas where the analytical formulas of the Airy disk, i.e. the uniform amplitude and phase aperture field distribution, facilitate the antenna design).

II. APERTURE FIELD METHOD TAILORED FOR OAM

A schematic of the antenna aperture field method as applied for the case of a conventional reflector antenna and an OAM antenna is shown in Fig. 1. The tangential aperture field of an OAM-carrying circularly-polarized (CP) field with a circularly-symmetric amplitude distribution $E(\rho')$ can be written as:

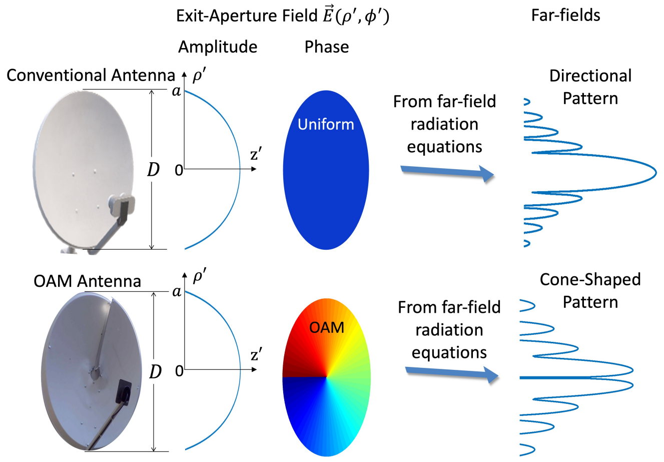


Fig. 1: Schematic of antenna aperture field method for conventional and OAM antennas.

$$\vec{E}_{\text{CP}}(\rho',\phi') = E(\rho')e^{-jl\phi'}(\hat{x} \pm j\hat{y})/\sqrt{2}, \quad 0 < \rho' < a \quad (1)$$

where ρ' and ϕ' are the radial and azimuthal coordinates in the cylindrical coordinate system; a is the transverse extend of the aperture field; $j = \sqrt{-1}$ is the imaginary unit; l is the OAM mode number; "$-$" and "$+$" signs in (1) indicate right-hand CP (RHCP) and left-hand CP (LHCP) waves, respectively. The equivalent magnetic current density is calculated from [2, eq. 6-129b]:

$$\vec{M}_s = (j\hat{x} \mp \hat{y})/\sqrt{2} \times E(\rho')e^{-jl\phi'}, \quad 0 < \rho' < a \quad (2)$$

The far-field integrals can be found using equations [2, eq. 6-125c, 6-125d]. The expression of far-field electric field is then calculated using [2, eq. 6-122b, 6-122c] through the far-field integrals in [2, eq. 6-125c, 6-125d]:

TABLE I:
Minimum theoretically achievable axial ratio (AR) of circularly-polarized OAM antennas at various beam peak elevation angles θ_c as predicted by (5).

θ_c	$\mathrm{AR_{min}}$	$\mathrm{AR_{min}}$ (dB)
0°	1.00	0 dB
15°	1.04	0.30 dB
30°	1.15	1.21 dB
45°	1.41	3.01 dB
60°	2.00	6.02 dB
75°	3.86	11.74 dB

$$\vec{E}^{ff}_{\mathrm{CP}}(r,\theta,\phi) = \frac{jk_0 e^{-jk_0 r}}{2r}\frac{(-j)^l}{\sqrt{2}} e^{\pm j\phi} e^{-jl\phi}\left(\hat{\theta} \pm j\cos\theta\hat{\phi}\right) I \tag{3}$$

where "$-$" and "$+$" signs in (3) correspond to RHCP and LHCP cases.
where

$$I = \int_0^a E(\rho')J_l(k_0\sin\theta\rho')\rho' d\rho' \tag{4}$$

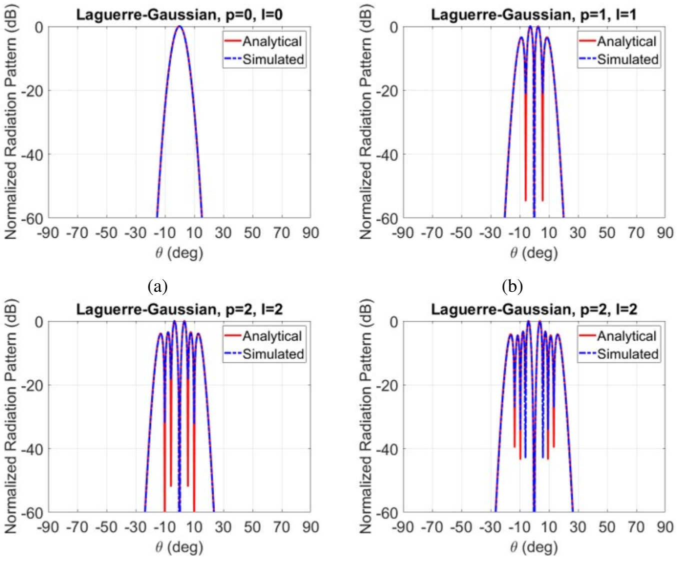
and $J_l(\cdot)$ is the l^{th}-order Bessel function of the first kind.

For a CP antenna, the axial ratio (AR) is a measure of its CP purity in the far field, and is defined as the ratio of the magnitudes of $\hat{\theta}$- and $\hat{\phi}$- components. The far-field expression (3) shows that for a CP OAM antenna, the AR at a given elevation angle θ_c can be obtained based on (3) as:

$$\mathrm{AR} = \frac{1}{\cos(\theta_c)} \tag{5}$$

This equation shows that the minimum theoretically achievable AR increases as the observing elevation angle increases. This is an important observation especially when considering the cone-shaped far-field pattern of an OAM beam [3]. Since the OAM far field in general peaks at an elevation angle that is non-zero (except for 0^{th} mode), the far field at the peak can no longer maintain the circular polarization purity, even with a perfectly pure CP aperture field. (5) serves as the baseline of achievable AR of a CP OAM communication system. As an example, the minimum theoretically achievable AR for an OAM CP antenna with a peak at $\theta_c \geq 45°$ would be greater than the common requirement of 3dB, as shown in Table I. The minimum achievable AR for different OAM peak angles is tabulated in Table I.

III. ILLUSTRATIVE EXAMPLE AND SIMULATION RESULTS



(a) (b)

(c) (d)

Fig. 2: Comparison between the analytical expressions (6)-(**??**) and the simulated data for the Laguerre Gaussian radiation pattern for different azimuthal and radial OAM mode numbers p, l, for $f = 19$ GHz and beam waist $w_g = 3.15\lambda$ (same as in reference [3]).

In this section, we present the full-wave simulation results to verify the observations made from the analytical formulas derived in previous sections. The Laguerre-Gaussian (LG) modes are chosen to be presented because they are one of the most popular example of OAM-carrying beams. In CST Microwave Studio, a planar near-field source containing the specified E-field given by [3, eq. 11] was used to represent the radiating aperture. The transient solver in CST was then used to calculate the far field of this aperture source. As an example, we modeled the Laguerre-Gaussian aperture distribution given by [3, eq. 11] for different azimuthal and radial OAM mode numbers p, l, for $f = 19$ GHz and beam waist $w_g = 3.15\lambda$, which is the same that was used in reference [3]. We also used the far-field expression (3) to calculate the far-field of the same LG beams:

$$\begin{aligned}&\vec{E}^{ff}_{\mathrm{LG}_{l,p}}(r,\theta,\phi)\\ &= \frac{jk_0 E_0^{\mathrm{LG}} e^{-jk_0 r}}{4\pi r}\left(\hat{\theta}\cos\phi - \hat{\phi}\cos\theta\sin\phi\right) w_g(-1)^p(-j)^l\\ &\times\sqrt{\frac{2\pi p!}{(p+|l|)!}}\left(\mathrm{sgn}(l)\frac{\psi}{\sqrt{2}}\right)^{|l|} e^{-\frac{\psi^2}{4}} L_p^{|l|}\left(\frac{\psi^2}{2}\right) e^{-jl\phi}\end{aligned} \tag{6}$$

where the same notation and variable definitions were used as in [3]. Excellent agreement between the simulated and analytically predicted patterns for the Laguerre-Gaussian distribution was achieved as shown in Fig. 2, thus validating our method of aperture field simulations in CST as well as the analytical expressions.

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