

# Evaluating Cross-Plane Acquisitions for Volume Process Tomography in the Laplacian Regime

Rafiul K. Rasel, Daniel Ospina Acero, Fernando L. Teixeira  
 Electrical and Computer Engineering  
 The Ohio State University  
 Columbus, OH 43212, USA  
 rasel.1@osu.edu, ospinaacero.1@osu.edu, teixeira.5@osu.edu

Qussai M. Marashdeh  
 Tech4Imaging LLC  
 Columbus, OH 43220, USA  
 marashdeh@tech4imaging.com

**Abstract**—Process tomography is a well established imaging modality to monitor a variety of flow processes in industrial applications. Traditionally, this has been done through imaging of a cross section of the domain. In recent years, much interest has been devoted to volume process tomography, where a three-dimensional reconstruction is directly obtained. However, depending on the sensor design, the number of independent measurements can be much higher in volume tomography compared to its two-dimensional counterpart. This makes the reconstruction problem more challenging and may prevent real-time monitoring in certain cases. In this work we investigate the optimal choice of cross-layer measurements to provide accurate volumetric tomography while minimizing image reconstruction costs using electrical capacitance volume tomography as example.

## I. INTRODUCTION

Process tomography is a widely used imaging modality to monitor multiphase flow processes in industrial and scientific applications [1], [2], [3], [4], [5], [6]. Although two-dimensional (2D) process tomography is relatively fast and robust, *volumetric* process tomography is highly desirable since it provides direct 3D information about the region of interest (RoI). However, depending on the sensor design, the number of independent measurements can be considerably higher versus its 2D counterpart. For example, an electrical capacitance tomography (ECT) sensor with 8 electrodes yields 28 independent measurements. On the other hand, a electrical capacitance volume tomography (ECVT) sensor with 10 layers and 8 electrodes in each layer yields 3160 independent measurements. Such large data set may exacerbate ill-conditioning and ill-posedness issues in the image reconstruction problem. An optimal sensing strategy that minimizes redundant or unnecessary measurements between different layers can help alleviate the costs and make volumetric process tomography faster and more robust [7]. One simple approach to generate volumetric images is by stacking 2D cross-sectional images obtained from different layers. Although effective for columnar or stratified flows, this strategy may not be always adequate for bubbly or more complex types of flows. Past works on cross-layer strategies for volumetric tomography are scarce [8], especially in the context of low-frequency applications with a Laplacian-like interrogating field. In this work, we investigate and evaluate sensing cross-layer strategies in the context of ECVT.

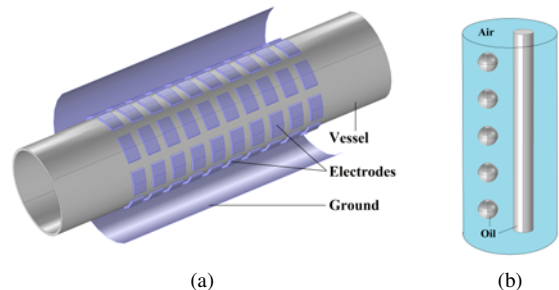


Fig. 1. (a) Comsol flow setup used for simulation, (b) Flow model used in the simulation with oil ( $\epsilon_r = 3$ ) drops and column dispersed in air ( $\epsilon_r = 1$ ) background.

## II. VOLUME PROCESS TOMOGRAPHY

### A. Background

Process tomography involves solving the forward problem and the inverse problem [9]. In the forward problem, sensors are used to obtain the measuring quantity, e.g. mutual capacitances between boundary electrodes in ECVT. The inverse problem seeks to reconstruct the desired quantity, e.g., permittivity distribution in the RoI in ECVT from the measurement data [10], [11]. Under a Born approximation [12] the forward problem writes as

$$[\mathbf{D}]_{M \times 1} = [\mathbf{S}]_{M \times N} [\mathbf{G}]_{N \times 1} \quad (1)$$

where  $[\mathbf{D}]_{M \times 1}$  is the measurement vector,  $[\mathbf{S}]_{M \times N}$  is the sensitivity (Jacobian) matrix [13], and  $[\mathbf{G}]_{N \times 1}$  is the vector of unknowns (permittivity voxel values). If the number of electrodes is  $M$ , there will be a total of  $n = \frac{M(M-1)}{2}$  independent mutual capacitance measurements. Since a direct inverse of sensitivity matrix  $[\mathbf{S}]_{M \times N}$  (e.g.,  $M \ll N$ ) does not exist, reconstruction techniques such as iterative Landweber (ILM) [14], with regularization strategies, needs to be implemented.

### B. Measurement Acquisition Strategies

The condition number of the matrix  $[\mathbf{S}]_{M \times N}$  for a conventional volumetric tomography sensors can be quite large. One way to improve the condition number is to eliminate unnecessary measurement across layers, such as one that are

widely separated in height. The question naturally arises as to which separation level does make sense to eliminate the measurements for a given ROI and sensor geometry. Fig. 2b shows an example with different choices in a sensor with 10 layers. The blue lines show inter-layer measurements that are kept for the image reconstruction and the red dotted lines show the measurements that are omitted. For a  $8 \times 10$  volumetric sensor (with 8 electrodes in each layer) there will be 3160 independent measurements in total if all electrode pair combinations are taken. By utilizing one and two neighbor electrode layers only (i.e., eliminating cross-layer measurements involving layers more than one or two layers apart) will result in 856 and 1358 measurements, respectively. Although the lack of space preclude a detailed discussion here, the decision criterion can be developed *a priori* for a given sensor and ROI geometry through an analysis of the condition number and the norm of the sensitivity matrix [15].

### C. Results

An ECVT sensor setup shown in Fig. 1a is considered here to illustrate the problem. The ECVT sensor has 10 layers with 8 electrodes in each layer. Two measurement schemes are shown in Fig. 2a and Fig. 2b. Fig. 2a corresponds to the traditional stacking of cross-section measurements to produce a volumetric image in 3D ECT [16]. Fig. 2b corresponds to a true ECVT configuration where cross-layer measurements are done as indicated by the blue lines. The red dashed lines correspond to discarded measurements (reduced ECVT). The condition number of the sensitivity matrix for 3D-ECT, reduced ECVT and full ECVT are 206.8,  $2.25 \times 10^4$  and  $1.18 \times 10^{10}$ , respectively. Although having the lowest condition number, 3D-ECT cannot resolve well the bubbly flow as illustrated in Fig. 2c where four drops are apparent instead of the five actual ones. It is clear from Fig. 2d that the reduced ECVT can reconstruct all five dispersed bubbles without any significant loss of resolution.

#### ACKNOWLEDGMENT

This work was supported in part by NASA under grant NNX16CC10C and by the DOE under grants DE-SC0011936 and DE-SC0010228.

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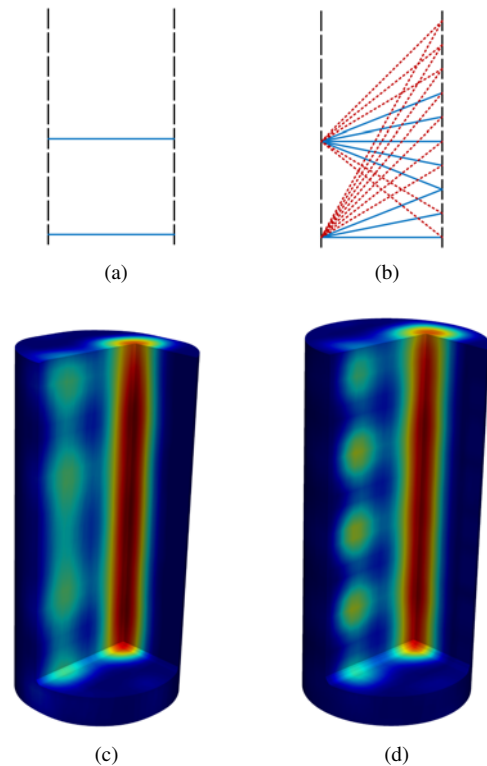


Fig. 2. (a) Measurement scheme for 3D-ECT, (b) measurement scheme for optimized ECVT, (c) reconstructed image of the flow model using 3D-ECT, and (d) reconstructed image of the flow model using optimized ECVT

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