Adaptive Sensing Matrix Design in Compressive Sensing Based Direction of Arrival Estimation with Hardware Constraints

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***Abstract*— Compressive sensing (CS) techniques are able to decrease hardware complexity in various applications including direction of arrival (DoA) estimation using antenna arrays. In CS based DoA estimation systems, analog outputs of the antenna elements are compressed by a matrix called sensing matrix and digitized after compression. This operation reduces the number of analog-to-digital converters in the hardware implementation. However, constraints resulting from hardware implementation of sensing matrices are not considered in general which can drastically increase the system complexity. In this study, we propose a novel adaptive sensing matrix design methodology by including such hardware implementation constraints.**

Keywords—Direction of arrival estimation, compressive sensing, sensing matrix design, hardware implementation

I. Introduction

Traditional direction of arrival (DoA) estimation methods like MUSIC and MVDR exploit the covariance matrix of the received signal which requires many snapshots and limits use of these methods to only uncorrelated sources. On the other hand, compressive sensing (CS) based DoA estimation techniques do not have these limitations and they can operate on compressed measurements. In other words, linear combination of the analog antenna element outputs can be digitized instead of digitizing each of them separately. Such linear combination is done by a matrix called sensing matrix which can be implemented with an analog beamforming circuit. Sensing matrix design is widely studied in the CS based DoA estimation literature. However, hardware complexity of the system is not generally considered comprehensively. Resulting designs require analog operations on every antenna element output before each digitizing channel which requires use of many low noise amplifiers (LNAs), attenuators and phase shifters. Even if this approach reduces the number of analog-to-digital converters (ADCs); huge number of LNAs, attenuators and phase shifters is undesirable from a hardware implementation perspective.

In radar systems, DoA estimation methods can be further improved by using the prior information generated by either the radar itself, or from other sources that may be available in the sensor network. This is especially desirable if there are different levels of importance assigned to different targets that are being tracked, which may be the case for most military radar applications. Most of the CS based sensing matrix design methods do not adapt to any prior information.

In this study, we propose a novel sensing matrix design methodology which can adapt to the target probability distribution in the environment, while drastically reducing the hardware complexity. Our design is applicable to uniform linear arrays (ULAs) with isotropic elements. We validate the performance improvement achieved by our method by using Monte Carlo simulations.

Upper-case bold and lower-case bold letters are used for matrices and vectors respectively. ℓ_1, ℓ_2, and Frobenius norms are denoted by $\|\boldsymbol{x}\|_1$, $\|\boldsymbol{x}\|_2$, $\|\boldsymbol{X}\|_\mathrm{F}$. $\boldsymbol{X}^\mathrm{T}$ and $\boldsymbol{X}^\mathrm{H}$ denote the transpose and the conjugate transpose of $\boldsymbol{X}$.

The proposed approach and numerical results are given in Section II and III respectively. Concluding remarks are given in Section IV.

II. Design Methodology

Let M, L, and m denote the numbers of antenna elements, grid points and digitized channels respectively. Then, signal model for a CS based DoA system is written as

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ is the received signal at a single snapshot, $\boldsymbol{A} = \frac{1}{\sqrt{L}}[\boldsymbol{a}(\theta_1)\ \boldsymbol{a}(\theta_2) \dots \boldsymbol{a}(\theta_L)]$ is the dictionary where $\boldsymbol{a}(\theta) = \left[1\ e^{j\omega(\theta)} \dots e^{j(M-1)\omega(\theta)}\right]^T$ and $\omega(\theta) = \frac{2\pi d}{\lambda}\cos(\theta)$ for a ULA with isotropic antenna elements. When the grid points θ_i's for $1 \le i \le L$ are chosen such that $\omega(\theta)$ values are uniformly separated, $\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{I}_M$ equality is achieved, which will be useful as will be shown later. $\boldsymbol{s} \in \mathbb{C}^{L\times 1}$ is the sparse source vector, $\boldsymbol{n} \in \mathbb{C}^{M\times 1}$ is an additive complex white Gaussian noise with standard deviation σ and mean $\boldsymbol{0}$, i.e., $\boldsymbol{n} \sim N_C(\boldsymbol{0}, \sigma^2\boldsymbol{I}_M)$. Introducing the sensing matrix $\boldsymbol{\Phi} \in \mathbb{C}^{m\times M}$, measurements are compressed as:

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{A}\boldsymbol{s} + \boldsymbol{\Phi}\boldsymbol{n}. \tag{2}$$

A whitening matrix $\boldsymbol{W}$ is applied to $\boldsymbol{\Phi}\boldsymbol{n}$:

$$\boldsymbol{W}\boldsymbol{y} = \boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{A}\boldsymbol{s} + \boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{n}, \tag{3}$$

where $\boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{n} \sim N_C(\boldsymbol{0}, \sigma^2\boldsymbol{I}_m)$. Finally, the following optimization problem is solved for DoA estimation:

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}} \|\boldsymbol{s}\|_1$$
$$s.t. \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{\Phi}\boldsymbol{A}\boldsymbol{s}\|_2^2 \le \beta^2, \tag{4}$$

where the $\beta^2 \cong \sigma^2 m$ is an appropriate choice for an unbiased comparison between different $\boldsymbol{\Phi}$'s as justified in [1].

Choice of $\boldsymbol{\Phi}$ is of crucial importance for reliable DoA estimation. Following sensing matrix design criterion is commonly used:

$$\widehat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \|(\boldsymbol{\Phi}\boldsymbol{A})^\mathrm{H}(\boldsymbol{\Phi}\boldsymbol{A}) - \boldsymbol{T}\|_\mathrm{F}^2, \tag{5}$$

For $\boldsymbol{T} = \boldsymbol{T}^\mathrm{H}$, which is reasonable to assume since $\boldsymbol{T}$ is the target Gram matrix, and for $\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{I}_M$, (5) is equivalent to:

$$\widehat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \|\boldsymbol{\Phi}^\mathrm{H}\boldsymbol{\Phi} - \boldsymbol{A}\boldsymbol{T}\boldsymbol{A}^\mathrm{H}\|_\mathrm{F}^2, \tag{6}$$

with the closed-form solution $\widehat{\boldsymbol{\Phi}} = \boldsymbol{\Sigma}_{Z_m}^{1/2}\boldsymbol{U}_{Z_m}^\mathrm{H}$ where $\boldsymbol{U}_{Z_m}\boldsymbol{\Sigma}_{Z_m}\boldsymbol{U}_{Z_m}^\mathrm{H}$ is the best rank-m approximation of $\boldsymbol{Z} \coloneqq \boldsymbol{A}\boldsymbol{T}\boldsymbol{A}^\mathrm{H} = \boldsymbol{U}_Z\boldsymbol{\Sigma}_Z\boldsymbol{U}_Z^\mathrm{H}$ [1], [2]. This design puts no restriction on the number of channels to which each antenna can connect. Therefore, it requires up to $m \times M$ phase

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shifters, attenuators and LNAs which drastically complicates the hardware implementation. This can be overcome by using a block diagonal $\mathbf{\Phi}$, i.e.:

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N \end{bmatrix}, \quad (7)$$

where N is the number of sub-matrices constituting $\mathbf{\Phi}$. Then:

$$\mathbf{\Phi}^{\mathrm{H}}\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_1^{\mathrm{H}}\mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2^{\mathrm{H}}\mathbf{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N^{\mathrm{H}}\mathbf{\Phi}_N \end{bmatrix}. \quad (8)$$

Naming the corresponding block diagonal parts of $\boldsymbol{Z}$ as $\boldsymbol{Z}_1, \boldsymbol{Z}_2, \dots, \boldsymbol{Z}_N$, i.e., elements of $\mathbf{\Phi}_n^{\mathrm{H}}\mathbf{\Phi}_n$ and $\boldsymbol{Z}_n$ have the same column and row indices, $\mathbf{\Phi}_n$'s are chosen as:

$$\widehat{\mathbf{\Phi}}_n = \mathbf{\Sigma}_{Z_{n_r}}^{1/2} \boldsymbol{U}_{Z_{n_r}}^{\mathrm{H}}, 1 \le n \le N, \quad (9)$$

where $\boldsymbol{Z}_n = \boldsymbol{U}_n \mathbf{\Sigma}_{Z_n} \boldsymbol{U}_{Z_n}^{\mathrm{H}}$ and $\boldsymbol{U}_{Z_{n_r}} \mathbf{\Sigma}_{Z_{n_r}} \boldsymbol{U}_{Z_{n_r}}^{\mathrm{H}}$ is the best rank-r approximation of $\boldsymbol{Z}_n$, r denoting the desired rank of $\mathbf{\Phi}_n$. Typically, r denotes the desired number of channels to which each antenna is connected, since $\mathbf{\Phi}_n$'s are typically wide and full row-rank matrices with r rows. Then, given $r < m$, much fewer number of LNAs, attenuators and phase shifters are used.

In (5), $\boldsymbol{T} = \boldsymbol{I}_L$ is chosen commonly, which is non-adaptive. An appropriate choice of $\boldsymbol{T}$ can lead to an adaptive design as in [1], [2]. Since we want to focus more on the regions where the probability of a target presence is higher, the following novel design criterion is proposed:

$$T_{ij} = \begin{cases} p(\omega_i), i = j \\ \quad 0, i \ne j \end{cases} \; 1 \le i, j \le L, \quad (10)$$

where $p(\omega_i)$ is the mixture probability mass function (PMF) of the targets in the environment at the grid point ω_i. For a target scene with Q targets, $p(\omega_i)$ can be written as the convex combination of individual target PMFs:

$$p(\omega_i) = \sum_{q=1}^{Q} \alpha_q p_q(\omega_i), 1 \le i \le L. \quad (11)$$

α_q's can be chosen, for example, based on the significance of the targets. In this study, $p_q(\omega_i)$'s are assumed to be known, for instance, from previous snapshots in practice.

III. Numerical Results

In our simulations, we use a ULA with $M = 36$ isotropic antenna elements that are separated by integer multiples of half-wavelength distance. 36 analog outputs of the antenna elements are digitized in $m = 12$ channels. Single snapshot is taken and $L = 180$ grid points are used. For the target scene, we assume that there are two targets having Gaussian DoA distributions with mean 70° and 120° respectively and both with standard deviation 2°. As a realistic approach, targets emerge on a continuous grid in our simulations. Signal-to-noise ratio is defined as $SNR \coloneqq 10\log_{10}(P_s/\sigma^2) + 10\log_{10} M$ where P_s denoting the target source power which is assumed to be the same for both targets. We choose $\alpha_1 = \alpha_2 = 0.5$ and $r = 2$, which means that both targets are of equal importance and each antenna element is connected to two channels only.

For comparison purposes, we use a (full) random Gaussian matrix and a random block diagonal matrix which has the form given in (7) with $\mathbf{\Phi}_n$'s are different random Gaussian matrices. Given that, hardware implementation of random Gaussian matrix requires $m \times M = 12 \times 36 = 432$ phase shifters, attenuators and LNAs since it is a full matrix. Physically, it means that each of $M = 36$ antenna elements is connected to all of $m = 12$ channels. When a random block diagonal matrix or the proposed design is used, this number drastically reduces to $r \times M = 2 \times 36 = 72$ since each antenna element is connected to 2 channels only. In addition to the $m/r = 6$ times difference between number of LNAs, attenuators and phase shifters, connecting that many components to each other is difficult especially for massive antenna arrays. The number of channels do not vary from design to design, hence 12 ADCs each. This quantitative analysis demonstrates that the random block diagonal matrix and the proposed design have the same hardware complexity, while the random Gaussian matrix leads to a much more complex and expensive architecture.

Simulating 50,000 Monte Carlo iterations each with different DoA and noise realizations, results given in Fig. 1 are achieved:

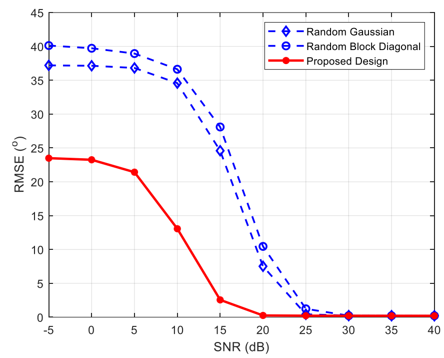

Fig. 1. Performance comparison of random Gaussian matrix, random block diagonal matrix, and the proposed method

As it is seen from Fig. 1, the proposed design outperforms others at all SNRs, even if it has much less hardware complexity compared to the random Gaussian matrix. We also see that random Gaussian matrix outperforms random block diagonal matrix. This is expected since random Gaussian matrix is a full matrix and connects each antenna element to all digitization channels. Interestingly, in some scenarios, block random Gaussian matrix outperforms random Gaussian matrix. We do not present these results here for brevity. However, it may be worth an investigation and one can refer to [3] for a theoretical foundation of restricted isometry property for random block diagonal matrices.

IV. Conclusions

In this study, we propose a novel adaptive sensing matrix design methodology with hardware implementation constraints and demonstrate its superior performance over random Gaussian and random block diagonal matrices.

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