MLFMA ANALYSIS OF FINITE APERTURE ARRAYS INCLUDING REFLECTOR

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Abstract **– An extended multilevel fast multipole algorithm (MLFMA) formulation is presented that includes magnetic surface current densities of radiating apertures and impedance relations containing modal excitation terms. The method-of-moment (MoM) solution is based on a combined field integral equation (CFIE) written in terms of mixed potential form. Aperture waveguide modal eigenvectors and Rao-Wilton-Glisson (RWG) functions for triangular patches are utilized for basis functions of the magnetic and electric surface current densities, respectively. The applicability of the method is demonstrated at the examples of a rectangular horn cluster in a finite screen and a finite circular horn cluster feeding a subreflector. Advantages and disadvantages of the MLFMA when applied for aperture array problems are discussed. The method is verified by available measurements and reference calculations.**

INTRODUCTION

Fast integral solvers based on the fast multipole method (FMM) [1] - [3], and more recently on the multilevel fast multipole algorithm (MLFMA) [1], [4] – [7], have proven being well appropriate for large-scale scattering problems. In the matrix equations for *N* unknowns resulting from the method-of-moment (MoM) solutions, the MLFMA allows matrix vector products being effected in O $(N \log N)$ operations [1], [4] – [7]. Applications of the MLFMA have mainly been restricted to scattering problems, so far. Due to its attractive efficiency potential, investigations of its applicability also to aperture radiating problems are desirable.

A MLFMA formulation for a single ridged horn has been discussed in [8]. First applications to rectangular apertures with an electric field integral equation (EFIE) approach have been introduced in [9]. This paper presents a combined field integral equation (CFIE) MFLMA solution with improved convergence for coupled rectangular or circular apertures in a finite screen, elucidates some advantages and disadvantages of the MLFMA at aperture radiating problems, and illustrates its applicability to a finite circular horn cluster feeding a subreflector.

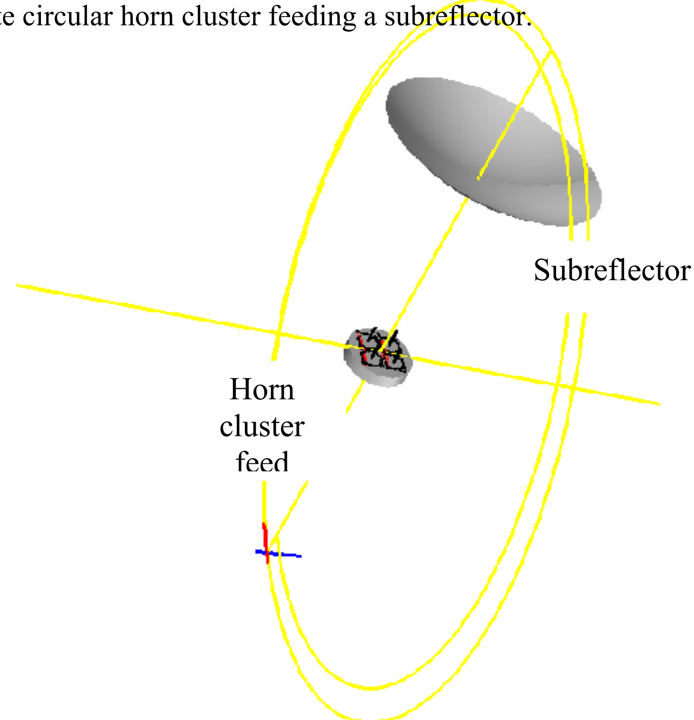

Fig. 1. Investigated example: Horns in cluster feed 15mm diameter, center distance 17.5mm, feed plate diameter 50mm, thickness 10mm. Hyperbolic subreflector 147.7 mm diameter, focus lengths 250mm, 40mm, frequency 24 GHz.

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THEORY

The EFIE formulation has been described in [9]. Hence, for the desired CFIE to increase the convergence behavior of the MLFMA solution, we still need the magnetic field integral equation (MFIE) part.

On the surface with unit normal vector $\hat{n}$ on finite structures of ideal conductivity with apertures (Fig. 1), the MFIE is formulated in terms of electric $\mathbf{J_s}$ and magnetic $\mathbf{M_s}$ surface current densities in the usual way

$$\mathbf{J}_s - \hat{n}\times\mathbf{H}^{\mathrm{J}}\left\{\mathbf{J}_s\right\} - \hat{n}\times\mathbf{H}^{\mathrm{M}}\left\{\mathbf{M}_s\right\} = \hat{n}\times\mathbf{H}^{inc} \quad , \tag{1}$$

where $\mathbf{J_s}$ are expanded in Rao-Wilton-Glisson (RWG) basis functions $\vec{f}_i$; and for $\mathbf{M_s}$ the normalized modal eigenvectors $\vec{g}_i = \mathbf{e}_i \times \hat{n}$ of the apertures are chosen [10]. The MFIE (1) is scalar multiplied by $\vec{f}_i$ and integrated over the corresponding area of the basis functions, as well as scalar multiplied by $\hat{n}\times\vec{g}_i$ and integrated over the aperture surfaces. This yields the linear equation system

$$\begin{pmatrix} \mathbf{T}_3 & \mathbf{Y}_1 \\ \mathbf{T}_4 & \mathbf{Y}_2 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{J} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{pmatrix} , \tag{2}$$

where $\mathbf{J}$, $\mathbf{b}$ are the vectors of corresponding expansion coefficients, and the matrix elements are given by

$$(\mathbf{T}_3)_{ij} = \iint_S \vec{f}_i \cdot \vec{f}_j dS + \iint_S (\hat{n}\times\vec{f}_i)\cdot\mathbf{H}^{\mathrm{J}}\left\{\vec{f}_j'\right\} dS \tag{3}$$

$$(\mathbf{T}_4)_{ij} = \iint_{S_{Ap}} \mathbf{e}_i \cdot \vec{f}_j dS + \iint_{S_{AP}} (\hat{n}\times\mathbf{e}_i)\cdot\mathbf{H}^{\mathrm{J}}\left\{\vec{f}_j'\right\} dS \tag{4}$$

$$(\mathbf{Y}_1)_{ij} = \iint_S (\hat{n}\times\vec{f}_i)\cdot\mathbf{H}^{\mathrm{M}}\left\{\vec{g}_i'\right\} dS \qquad (\mathbf{Y}_2)_{ij} = \iint_{S_{Ap}} (\hat{n}\times\mathbf{e}_i)\cdot\mathbf{H}^{\mathrm{M}}\left\{\vec{g}_i'\right\} dS \tag{5}$$

$$(\mathbf{V}_1)_i = -\iint_S (\hat{n}\times\vec{f}_i)\cdot\mathbf{H}^{\mathbf{inc}} dS \qquad (\mathbf{V}_1)_i = -\iint_{S_{Ap}} (\hat{n}\times\mathbf{e}_i)\cdot\mathbf{H}^{\mathbf{inc}} dS \tag{6}$$

For the MLFMA, the matrices are separated according to the principle [3] – [9]

$$\mathbf{T},\mathbf{Y} = \mathbf{T}^{MoM},\mathbf{Y}^{MoM} + \mathbf{T}^{FMM},\mathbf{Y}^{FMM} \quad , \tag{7}$$

where the 'near-neighbor' parts $\mathbf{T}^{MoM}$, $\mathbf{Y}^{MoM}$ are solved directly by the standard MoM (after having extracted the singularity in the known way) with integrations only along the near neighbor range [3] – [7]. For the 'far-neighbor' parts $\mathbf{T}^{FMM}$, $\mathbf{Y}^{FMM}$ the MLFMA yields for the matrix $\mathbf{T}_3$ analogue expressions to known formulations at scattering problems; for $\mathbf{T}_4^{FMM}$, we obtain

$$(\mathbf{T}_4^{\mathbf{FMM}})_{ij} = +\frac{jk}{4\pi}\frac{jk}{4\pi}\sum_m \iint T_\infty(kR;\hat{k}\cdot\hat{R}) \iint_{S_{im}} e^{-jk\hat{k}\cdot\vec{r}_m}(\hat{n}\times\mathbf{e}_{\mathbf{im}})dS\times\hat{k} \;\cdot\; \iint_{S_j} e^{+jk\hat{k}\cdot\vec{r}_{m'}}(\mathbf{I}-\hat{k}\hat{k})\cdot\vec{f}_j' dS' dA_{\hat{k}} \;, \tag{8}$$

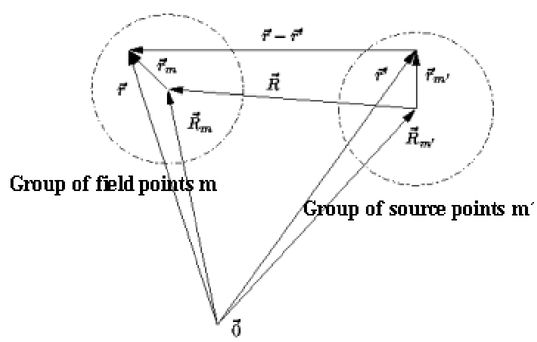

Fig. 2. Geometrical relations for source and field points used in the MLFMA equations

with the abbreviation

$$T_\infty\left(\kappa;\cos\alpha\right) = \sum_{l=0}^{\infty}\frac{2l+1}{j^l}\,\mathrm{h}_l^{(2)}(\kappa)\,\mathrm{P}_l(\cos\alpha) \quad , \tag{10}$$

where P are the Legendre polynomial, and $\mathrm{h}^{(2)}$ the spherical Hankel function of 2^{nd} kind. The geometrical relations are elucidated in Fig. 2, k and Z_F are the free space wavenumber ($\hat{k}$ the corresponding vector) and the free space wave impedance, respectively.

The expressions for **Y** in (2) are based on a mixed potential formulation. The near-neighbor part is again given by standard MoM (singularities extracted); for the far-neighbor part $\mathbf{Y}_1^{FMM}$, we obtain

$$(\mathbf{Y}_1^{\mathrm{FMM}})_{ij} = +\frac{jk}{4\pi Z_F}\frac{jk}{4\pi}\sum_{m'}\iint T_\infty(kR;\hat{k}\cdot\hat{R})\times\iint_{S_i} e^{-jk\hat{k}\cdot\vec{r}_m}(\hat{n}\times\vec{f}_i)dS \;\cdot\; \iint_{S_{jm'}} e^{+jk\hat{k}\cdot\vec{r}_{m'}}(\hat{k}\times\vec{g}'_{jm'})dS'dA_{\hat{k}}\,. \tag{11}$$

Similar expressions are found for $\mathbf{Y}_2^{FMM}$.

The CFIE is formulated as the linear combination of the EFIE with the MFIE using the parameter α [1]

$$\left(-\frac{4\pi\alpha}{jkZ_F}\begin{pmatrix}\mathbf{Z}_1 & \mathbf{T}_1\\ \mathbf{Z}_2 & \mathbf{T}_2\end{pmatrix}+\frac{4\pi(1-\alpha)}{jk}\begin{pmatrix}\mathbf{T}_3 & \mathbf{Y}_1\\ \mathbf{T}_4 & \mathbf{Y}_2\end{pmatrix}\right)\cdot\begin{pmatrix}\mathbf{J}\\ \mathbf{b}\end{pmatrix} = -\frac{4\pi\alpha}{jkZ_F}\begin{pmatrix}\mathbf{U}_1\\ \mathbf{U}_2\end{pmatrix}+\frac{4\pi(1-\alpha)}{jk}\begin{pmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{pmatrix}\,. \tag{12}$$

In addition to the field integral equations, we still have to formulate the impedance relations on the waveguide apertures. From the continuity of the tangential magnetic field strength on the apertures

$$2\mathbf{H}_\mathrm{t}^{\mathrm{inc}} + \mathbf{H}_\mathrm{t}^{\mathrm{r}}\left(\mathbf{M}_\mathrm{S}\right) = \mathbf{J}_\mathrm{S}\times\hat{n}\,, \tag{13}$$

and with the sum expressions containing the modal eigenvectors **h** of the forward directed (p) and reflected (r) wave terms

$$2a_i\mathbf{h}_i^p + \sum_j b_j\mathbf{h}_j^r = \sum_k \mathbf{J}_k\vec{f}_k\times\hat{n} \tag{14}$$

we obtain with $\mathbf{h}_i^p = -\mathbf{h}_i^r$, after scalar multiplication with $\vec{g}_l = \mathbf{e}_l^r\times\hat{n}$ and integration over the corresponding aperture areas,

$$\sum_k J_k\iint\mathbf{e}_l^r\cdot\vec{f}_k dS - \sum_j b_j\iint(\mathbf{e}_l^r\times\hat{n})\cdot\mathbf{h}_i^r dS = -2a_i\iint(\mathbf{e}_l^r\times\hat{n})\cdot\mathbf{h}_i^r dS\ . \tag{15}$$

This impedance relation contains the modal excitation term for an excitation with the *i*-th waveguide mode. The equation is correspondingly weighted and added to the lower part of the EFIE.

RESULTS

The first example are three radiating rectangular apertures according to Bird [11] but within a finite plate, shown in Fig. 3. For the MLFMA, 7105 unknowns, 336 groups, 3 levels have been considered. A Cholesky pre-conditioner has been applied. The storage requirement was 180 MB.

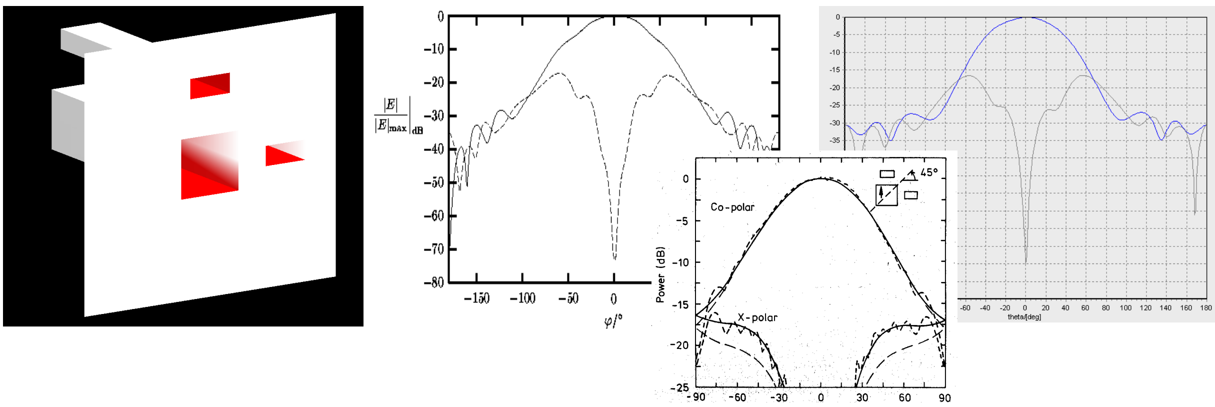

Fig. 3. Three rectangular apertures according to [11], but in finite rectangular plate of size 100mm x 100mm. Aperture sizes: 22.8mm^2 center aperture, 15.7mm x 7.7mm lateral apertures, displacement 30mm, f = 12.5 GHz

Good agreement with measurements provided in [11] up to +/- 90° can be stated. In comparison with own MoM reference calculations (up to +/- 180°), however, it is evident that relative field values of around –30dB are about the limit at this array radiation example that can reliably be resolved by the MLFMM.

To show the applicability for more complicated examples, a horn cluster consisting of four circular apertures in a finite circular screen together with a subreflector (Figs. 1, 4) is chosen. All apertures are assumed being excited with their fundamental modes, with same amplitude and phase. For the MLFMA, 12772 unknowns, 1616 groups, 4 levels have been considered. 6 modes are taken into account in each aperture. The storage requirement was 365 MB.

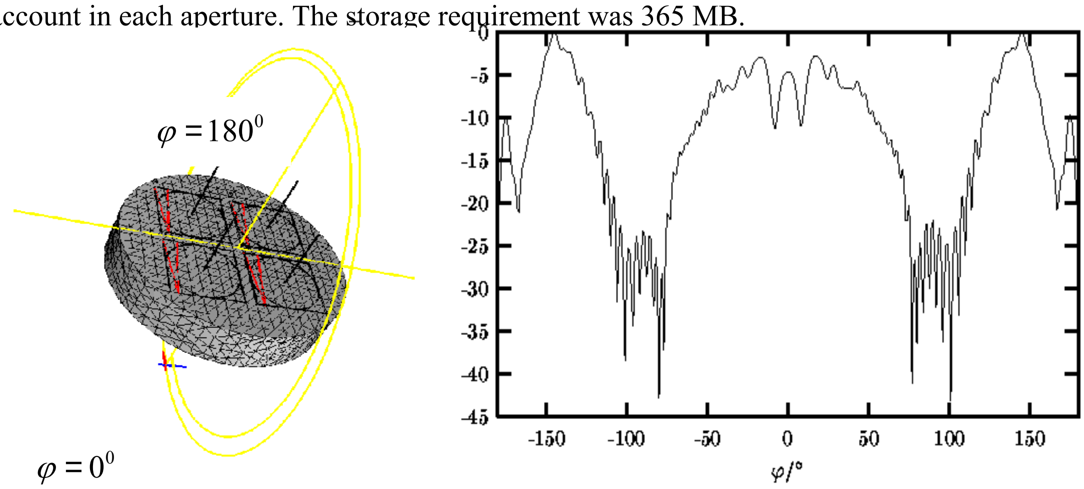

Fig. 4. Four circular apertures in a finite circular metallic screen of diameter 50mm and thickness 10mm together with a subreflector (Fig. 1). Frequency 24 GHz.

CONCLUSION

An extended multilevel fast multipole algorithm (MLFMA) for finite aperture arrays has been described. The CFIE formulation written in terms of mixed potential form and the application of aperture waveguide modal eigenvectors and Rao-Wilton-Glisson (RWG) functions for triangular patches as basis functions for the magnetic and electric surface current densities, respectively, yield stable and convergent results. The MLFMA leads to a significant reduction in required storage capacity; however, the resolution limit of about –30dB observed at a rectangular horn array could be problematic for this kind of applications, in particular when accurate cross-polarization predictions are required.

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