Leaky Surface-Plasmon Theory for Dramatically Enhanced Transmission through a Subwavelength Aperture,

Part I: Basic Features

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I. Introduction

About three years ago, several groups in the physics community working collaboratively found experimentally a very interesting and initially puzzling effect. For a couple of years before that they had been studying a ***periodic array*** of ***subwavelength holes*** in a metal plate at optical wavelengths. They reported that under the right conditions the transmission through the holes increased dramatically. More recently, they have been investigating the transmission through a ***single*** subwavelength-sized hole in a metal plate. When this tiny hole is made in a smooth metal plate the transmission through the hole was extremely small, as expected from small-aperture theory. Then, they found experimentally that when a ***periodic array*** of dimples or grooves is imposed on the initially smooth surface surrounding the hole the transmission of power through the hole is enhanced by "up to several orders of magnitude" [1].

This astonishing increase in the transmission was difficult to believe at first, but measurements at optical frequencies were made at several laboratories and they have been reported in many papers. There is general agreement that surface plasmons (a class of surface waves) play a key role in this enhancement in transmission, but, to quote one recent paper [2], "Although surface plasmons clearly play an important role in the transmission enhancement, the details of the enhancement mechanism are not yet understood." We believe that this interesting effect can be fully explained in terms of ***leaky waves***, and we present here, in Part I, the main features of the appropriate leaky-wave theory. In the companion paper, Part II, a leaky-wave antenna model is analyzed that quantitatively demonstrates the fundamentals of the effect.

All of the earlier studies were based on imposing around the tiny hole a periodic array of dimples or grooves on ***only one face*** of the metal plate, the one facing the incident light, and the other face was left smooth. The stress in those studies was on the transmission enhancement only. Furthermore, the radiation on the exit face, emerging as it did from a tiny hole, produced a very wide beam. The latest study [3] placed a periodic array of grooves on ***both*** faces. It was found that the transmission enhancement was further strongly increased, and that the radiation from the exit face was in the form of a ***narrow*** collimated beam, with a 3 dB beamwidth of about $\pm 5^{\circ}$ in their case, rather than the very wide beam obtained with a smooth face. To quote the paper [3], "Perhaps the most non-intuitive aspect of this phenomenon is the fact that scattering at the grooves of only a small fraction of the light emerging from the aperture can create such a narrow beam by interference." Putting aside the words "by interference," which are incorrect, our leaky-wave theory explains all the aspects of this investigation quite simply and completely, and, even further, tells one how to improve the design of the periodic structure.

II. The Properties of a Typical Structure

The metal film is generally made of silver, and the periodic structure imposed on the surface may be in the form of a square or rectangular array of indentations like dimples, or a radial array of circular grooves. For simplicity, measurements have also been taken on a narrow slit rather

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than a small hole, with an array of parallel grooves on each side of the slit. The wavelength appropriate for the measurements depends on the period of the array.

A sketch of a typical structure with grooves on both the entrance and exit faces is shown in Fig.1. For now, let us ignore the blue horizontal arrows, which relate to the leaky-wave theory. Using the terminology and the numbers from Fig. 2 of Lezec et al. [3], we have that period a = 500 nm, thickness h of the silver film = 300 nm, slit width = 40 nm and groove depth = 60 nm. We will return to Fig. 1 later.

Metals at optical wavelengths generally have a negative dielectric constant, and silver has the additional virtue of relatively low loss. The dielectric constant is strongly frequency dependent, according to the well-known relation

$$-\varepsilon_p = |\varepsilon_p| = (\omega_p/\omega)^2 - 1 \tag{1}$$

when losses are neglected, and where the subscript p means "plasma," since the metal at optical frequencies behaves like an overdense plasma. The term ω_p represents the plasma frequency, and $\omega_p > \omega$. The measurements in [3] show that the wavelength for a radiated beam at broadside is about 600 nm. At this wavelength, the value of the dielectric constant ε_p for silver is approximately -13. Knowledge of ε_p is important if we wish to employ the leaky-wave theory to optimize the design of the structure.

It is well known that the interface between air and an overdense plasma (corresponding to a negative dielectric constant) can support a surface wave, known as a ***surface plasmon*****,** that decays transversely away from the interface on both sides, but more rapidly on the plasma side. For this reason, the entrance and exit faces of the thin silver film can be viewed as independent of each other, and are connected only by means of the tiny hole between them.

The published papers on this enhancement effect have recognized that a surface plasmon will be present and will interact with the periodic structure. These papers are unclear and incomplete with respect to the actual mechanism of the interaction, and they use phrases such as "interference," or "Bragg-scattered to form a standing wave." The leaky-wave explanation shows clearly that the wave is a traveling wave that radiates at broadside, as will be seen.

A simple derivation for the dispersion behavior of the surface plasmon on a smooth interface shows that

$$\beta^P/k_0 = \left[|\varepsilon_p|/(|\varepsilon_p| - 1)\right]^{1/2} = \left[\{(\omega_p/\omega)^2 - 1\}/\{(\omega_p/\omega)^2 - 2\}\right]^{1/2} \tag{2}$$

where β^P is the phase constant of the surface plasmon and k_0 is the free-space wavenumber in air. Figure 2 presents the dispersion behavior in the form of a band-structure (or Brillouin) diagram, which plots k_0a vs. βa, where a is the period of the periodic structure on the surface. This figure will be key to explaining the leaky-wave theory, but for now we should look only at the red (heavy-line) curve and at the dashed 45° line from the origin.

At low frequencies β^P is seen from (2) to be almost the same as k_0, so that the dispersion (red) curve hugs the 45° line. As frequency increases, $|\varepsilon_p|$ decreases and the curve begins to move away from the 45° line. For silver at our frequencies of interest the dispersion curve is distinctly away from the 45° line but not greatly so. At still higher frequencies the curve moves rapidly away from the 45° line and becomes horizontal, with β^P approaching infinity when $(\omega_p/\omega)^2 = 2$ or $|\varepsilon_p| = 1$. This portion of the dispersion curve is off the scale of this plot.

III. The Leaky-Wave Theory for the Enhancement Effect

We first summarize the key steps in the theory of leaky-wave radiation from open periodic structures, making use of Fig. 2. A basic paper in 1959 [4] was the first to demonstrate that a surface wave guided along a periodically modulated surface would turn into a leaky wave when the frequency is raised sufficiently. This leaky wave then radiates power away at an angle determined by the period, the wavelength and the dispersion properties of the surface wave.

In Fig. 2, we first note the presence of the yellow-filled triangles, within which all solutions are real (except very near the edges). One can also show readily that outside of these triangles all solutions must be complex, corresponding physically to leaky waves with phase constant β^{LM} and leakage (attenuation) constant α^{LM}. The plot in Fig. 2 exhibits only the behavior of β^{LM}. We next note the presence of the red curve, which represents the dispersion curve for the n = 0 space harmonic of the surface plasmon. Inside the first triangle the n = -1 backward space harmonic crosses the n = 0 curve at $\beta^{LM}a = \pi$ to produce a (Bragg) stop band (only the crossing is shown). As the frequency is raised somewhat, the n = 0 dispersion curve crosses the right-side edge of the first triangle. At that frequency the n = -1 forward space harmonic begins to radiate in the backward endfire direction, and then that radiated beam swings up from backward endfire toward broadside as the frequency is increased further. When

$$\beta^{LM}a = 2\pi \quad \text{or} \quad \lambda_g = a \tag{3}$$

where λ_g is the guide wavelength (= $2\pi/\beta^{LM}$), the beam points at broadside itself. Then, a further increase in frequency causes the beam to enter the forward quadrant.

The red dispersion curve in Fig. 2 holds for the leaky surface plasmon with the periodic structure present. In the absence of the periodic structure, the dispersion properties will change somewhat, and the red curve would then represent β^P instead of β^{LM}. If β^P is used in (3), the radiated beam will not point precisely at broadside, but will consist of two beams at slightly different angles on each side of broadside, one beam from each side of the periodic array on the exit face. This effect is demonstrated numerically in Fig. 5 of the companion paper, Part II.

Let us next return to Fig. 1, and note the ***red arrows*** representing the incident wave on the entrance face and the narrow radiated beam on the exit face. Let us then consider what happens at the exit face when the power passing through the slit or hole emerges. Part of the power radiates directly into space, but the associated radiation pattern will be very broad. Another part of the power goes into a surface plasmon that gets excited on the exit face surrounding the hole. If the surface plasmon becomes a leaky wave, the amplitude of the leaky wave decreases as it travels away from the hole, as shown by the ***blue arrows*** of decreasing length appearing in Fig. 1.

We see therefore that the leaky wave effectively gathers up almost all of the power in the surface plasmon and radiates it into the broadside direction. To achieve a narrow beam at broadside, condition (3) must be satisfied with β^{LM} corresponding to the phase constant of the leaky plasmon as influenced by the periodic structure (not that of the surface plasmon on the smooth surface). For optimum efficiency, the value of α^{LM} must “match” the number of periods in the array, that is, α^{LM} must be such that about 90% or so of the power in the leaky wave must be leaked away before the leaky wave reaches the end of the periodic array. This consideration is customary in the design of leaky-wave antennas.

In the companion paper, Part II, Fig.3 shows that a very narrow beam can indeed be achieved, with a peak more than 20 dB higher than the value calculated for the direct radiation when the periodic structure is absent. The measured radiation patterns reported in [3] cover only the top 10 dB from the peak, so that it is very likely that the measured patterns in [3] consist almost entirely of the radiation from the leaky plasmon alone.

On the entrance face the process is the reverse of that described above for the exit face. The power entering the hole due to the normally incident plane wave is due not only to the direct power channeled into the slit or hole, but also to the leaky plasmon that collects power from the area of the periodic structure. It is understandable, therefore, that the power going out of the hole on the exit face is greatly enhanced by the periodic structure on the entrance face. When the exit face also possesses the periodic array, the exit beam is very narrow (due to the radiating leaky wave on the exit face), so that the radiated power density in the broadside direction is further enhanced, agreeing with the experimental observation that the overall enhancement in the transmission, as compared with that when the faces are smooth, is "several orders of magnitude."

References

[1] H. J. Lezec, T. W. Ebbesen, E. Devaux, A. Degiron, F. J. Garcia-Vidal, L. Martin-Moreno and R. A. Linke, "Overcoming Optical Diffraction from a Sub-Wavelength Aperture," *PIERS 2002*, p. 649, Cambridge, MA, July 2002.

[2] T. Thio, "Giant Light Transmission through Subwavelength Apertures in Metal Films," *PIERS 2002*, p. 648, Cambridge, MA, July 2002.

[3] H. J. Lezec, A. Degiron, E. Devaux, R. A. Linke, L. Martin-Moreno, F. J. Garcia-Vidal, and T. W. Ebbesen, "Beaming Light from a Subwavelength Aperture," *Science*, Vol. 297, pp. 820-822, 2 August 2002.

[4] A. A. Oliner and A. Hessel, "Guided Waves on Sinusoidally Modulated Reactance Surfaces," *IRE Transactions on Antennas and Propagat.*," Vol. AP-7, pp. S201-S208, December 1959.

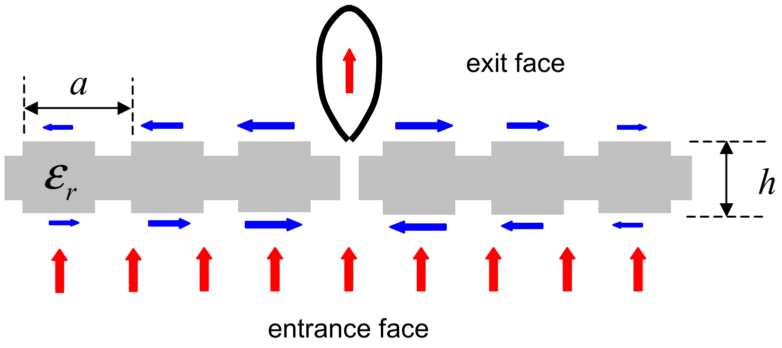

Fig. 1. Sketch of a metal film with a central small hole or slit and with a periodic array of grooves around it on both faces of the film. The vertical (red) arrows indicate the incident and transmitted light, and the horizontal (blue) arrows represent the variation in the amplitude of the leaky surface plasmon.

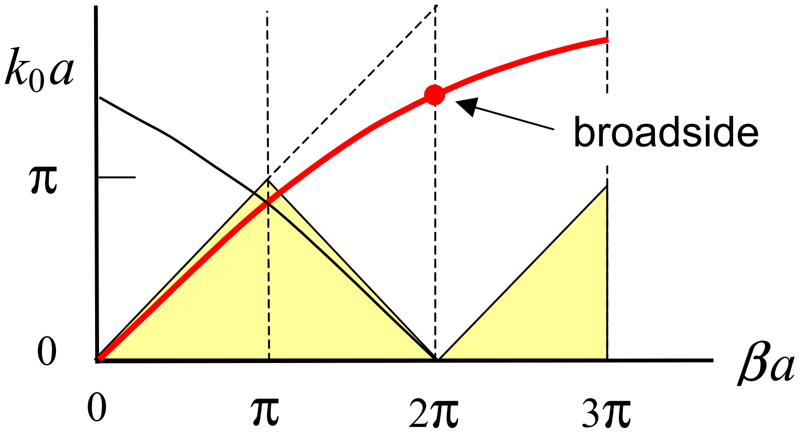

Fig. 2. Band-structure (or Brillouin) diagram used to explain the variation of the phase constant β^{LM} of the leaky surface plasmon as a function of the free-space wavenumber k_0. The heavy-line red curve represents the dispersion curve of the surface plasmon, which becomes leaky after it emerges from the triangles.