BACKARD WAVE PROPAGATION IN WAVEGUIDE FILLED WITH NEGATIVE PERMEABILITY META MATERIAL

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ABSTRACT: *The waveguide filled with lossles dispersionless material with negative permeability is analyzed in this paper. It is shown that such a waveguide supports propagation of the backward waves below the cut-off frequency. There is no propagation above the cut-off frequency and waveguide behaves as a low-pass filter, thus, it is a dual of conventional waveguide. Theoretical analysis was verified by measurements of scattering coefficients of rectangular waveguide loaded with split ring resonators in 10 GHz frequency band. As expected, the propagation band occurred below the cut-off frequency. It was observed that increase of the frequency caused the increase of electrical length of the waveguide. Therefore, the Foster's theorem is satisfied for this waveguide. It was also shown that the increase of the physical length of the waveguide causes the decrease of the electrical length. This unusual behavior is consequence of the fact that phase of the backward wave leads along the waveguide.*

1. INTRODUCTION

The propagation of the electromagnetic wave in lossless material with simultaneously negative permeability and permittivity ('left-handed' or 'backward' material) was analyzed for the first time back in 1968 [1]. Since such a material does not exist in nature, this analysis was considered as a pure theoretical curiosity until the introduction of artificial backward material in 2000 [2]. The backward material supports propagation of backward waves, thus, direction of phase velocity is opposite to the direction of energy flow. This is accompanied with reversal of some basic electromagnetic phenomena such as Snell law and Doppler effect. Some potential applications of this material such as high resolution electromagnetic lenses [3], subwavelength resonator [4] and gain enhancement of a small linear antenna [5] have been already proposed.

So far, electromagnetics of backward materials have been investigated experimentally in free-space [2], planar structures [6] and in waveguide environment [7]. Very recently [8], the propagation below cut-off frequency of a waveguide loaded with split-ring resonators was demonstrated. This phenomenon was interpreted by thinking of the waveguide below cut-off frequency of dominant TE mode as the one-dimensional material with negative effective permittivity [8]. Therefore, the waveguide loaded with split-ring resonators was interpreted as a material with both negative permeability and negative permittivity. Although the experimental results in [8] show the propagation bellow the cut-off, it was not attempted to verify the existence of the backward waves.

In this paper, the original idea [8] is analyzed and physical interpretation is proposed. Several waveguides have been designed, fabricated and measured. The propagation of the backward waves bellow cut-off has been shown experimentally .

2. THEORETICAL ANALYSIS

Consider a rectangular waveguide filled with arbitrary lossless and dispersionless isotropic material. The propagation factor of this waveguide is given by well known expression :

$$k = \pm \quad 2\pi f \sqrt{\varepsilon_0 \mu_0 F}, \quad F = \varepsilon_r \mu_r \left[1 - \frac{1}{(\varepsilon_r \mu_r)^2} \left(\frac{f_c}{f} \right)^2 \right] \quad . \tag{1}$$

Here, k stands for waveguide propagation factor, ε_0 stands for free-space permittivity and μ_0 for free-space permeability. The relative permittivity and permeability are denoted by ε_r and μ_r, respectively. Symbol f_c represents the waveguide cut-off frequency and f is the frequency of signal. F is a function which describes waveguide dispersion. In ordinary waveguide filled with material with positive permeability and permittivity one chooses the positive sign in front of square root in (1). This yields very well known high-pass behavior (Fig. 1a). Above the cut-off frequency, the propagation is in the form of the forward waves. Below the cut-off frequency, k becomes imaginary number and there is no propagation (stop-band) due to evanescent field inside the waveguide. It has been shown experimentally [7] and by computer simulation [10] that propagation is also possible in a waveguide filled with material with simultaneously negative permittivity and permeability (Fig. 1b). The propagation is in the form of

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backward waves and one must choose the negative sign in front of square root in (1). Generally, choosing the proper sign in front of square root in (1) is difficult task. One should check the physical validity of solutions of Maxwell curl equations taking into account possible different combinations of signs of permittivity and permeability.

In cases of the waveguide filled with ordinary or backward material, the sign of F (and therefore the location of stop-band) is independent on signs of ε_r and μ_r (since $\varepsilon_r\mu_r>0$). The case with opposite signs of the permittivity and permeability is more complicated and has not been analyzed extensively so far. Here, the sign of F depends both on the sign of the most right term in (1) and on the sign of ε_r and μ_r. It makes the choice of sign of square root even more difficult.

If one first assumes an empty waveguide ($\varepsilon_r=1$, $\mu_r=1$), the expression under the square root in (1) becomes the product of ε_0 and the function F. Formally, one can think of product $\varepsilon_0 F$ being the permittivity function and interpreting F as effective permittivity function. Following this way of thinking, the effective relative permittivity was defined [8] as:

$$\varepsilon_{reff} = 1 - \left(f_c / f\right)^2 \tag{2}$$

The effective relative permittivity has negative sign below the cut-off frequency and positive sign above it. It is important to find a physical interpretation of (2). Of course, it is clear that the relative permittivity or relative permeability of some material does not change when such a material is put into the waveguide. Equation (2) may be interpreted as the relative permittivity of fictitious one-dimensional material (or transmission line) having the same transmission properties as the waveguide. From the basic electromagnetic theory it is very well known that one can resolve field in a waveguide into two plane waves. These plane waves impinge the waveguide walls at different angles at different frequencies. For each plane wave, the component of the Poynting vector which is normal to the waveguide walls experiences total reflection. It causes the standing wave in transversal direction, existence of which is needed in order to satisfy boundary conditions. The component of Poynting vector which is parallel to the waveguide walls contributes to the propagation of the energy along the waveguide. Thus, the propagation along the waveguide is determined by either **E** field of the plane wave and transversal component of **H** field in TE mode or vice versa in TM mode. Taking the ratio of components of **E** and **H** field which contributes to the propagation leads to well known definition of the wave impedance (Z_w) in TE mode:

$$Z_w(f>f_c)=\frac{+\sqrt{\mu_0\mu_r/\varepsilon_0}}{\left|\sqrt{1-(f_c/f)^2}\right|},\quad Z_w(f<f_c)=+j\frac{+\sqrt{\mu_0\mu_r/\varepsilon_0}}{\left|\sqrt{1-(f_c/f)^2}\right|}. \tag{3}$$

In the derivation of (3) it was assumed that the waveguide is filled with magnetic material. The imaginary character of wave impedance below the cut-off is usually interpreted as inductance, hence, the waveguide acts as an inductive storage element. This can be explained by thinking of the waveguide as an ordinary two wire transmission line loaded with infinite number of short-circuited stubs. Below the cut-off frequency, the stubs are shorter than $\lambda/4$, thus they have inductive character. But, one can also think of imaginary character of Z_w as being the negative capacitance. So, one can rewrite the equation for wave impedance with introduction of effective relative permittivity (2) which may take negative values:

$$Z_w=\frac{\sqrt{\mu_0\mu_r/\varepsilon_0}}{\sqrt{1-(f_c/f)^2}}=\sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_{reff}}}. \tag{4}$$

Now one can investigate the influence of negative permittivity material on propagation. If μ_r has negative sign it will compensate the negative sign of ε_{reff} below the cut-off frequency. This leads to real character of Z_w, therefore, the propagation becomes possible. Now, one should choose the negative sign in (1), therefore, propagation is possible in the form of backward waves. So, the net effect is the same as it would be in the case of material with negative permittivity and negative permeability. Above the cut-off frequency, Z_w has imaginary character (negative inductance caused by negative permeability) and there is no propagation. This waveguide would behave as a low-pass filter (Fig. 1c), thus, it could be considered as a dual of conventional waveguide.

The analysis presented so fare dealt with (nonphysical) dispersionless material. In reality, any passive material has dispersion. Moreover, the derivatives of the products $f\mu_r$ and $f\varepsilon_r$ in respect to f should be positive [10], due to energy conservation. It can be shown that these restrictions do not change

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the basic conclusion that it is possible to have propagation bellow the cut-off frequency in the form of the backward waves. The important difference is that waveguide may not act as a high-pass filter. The propagation will occur only in frequency band where signs of meta material permeability and waveguide effective permittivity are both negative. So, for the meta material with resonant inclusions [2,5,7,8] the waveguide will behave as a band pass filter with central frequency located below the waveguide cut-off frequency.

3. EXPERIMENTAL RESULTS

In order to verify theoretical analysis, several prototypes of waveguide were developed and tested. The first waveguide was manufactured out from copper and it has length of 90 mm and cross section of 9 mm x 9 mm (Fig. 2a). Two flanges which fit the standard X-band waveguide were soldered at the ends of the waveguide. The split ring resonators ([2,8]) were designed to have resonant frequency at 10 GHz. The dimensions of the resonators are: the width of all tracks of 0.7 mm, inner radius of 1.5 mm, outer radius of 2.45 mm, the width of the gap between the rings of 0.25 mm and width of the slit of 0.7 mm. The resonators were fabricated on the ComClad substrate (thickness 0.5 mm, ε_r=2.6). First, it was attempted to clear some doubts in interpretation of the experimental results published in [8]. One could speculate that waveguide loaded with the split ring resonators actually acted as some kind of 'coaxial transmission line' without propagation of backward waves. The 'central conductor' of this line could be interpreted as a chain of magnetically coupled loop antennas (split ring resonators). It could have been possible due to the fact that first and last resonators shown in Fig. 3 in [8] extended beyond the waveguide and were located close to the probes of waveguide adapters. In order to clear this point, the only one row comprising 20 resonators with lattice constant of 5 mm was put centrally into the waveguide (Fig. 2a). Note that first and last resonators do not extend beyond the waveguide. The waveguide loaded with resonators was interfaced with two standard X-band waveguides and S_{21} parameter was measured using HP8720B network analyzer. The results are shown in Fig. 3. It can be seen that pass-band appeared below the cut-off, at the frequency of 10 GHz. Thus, the hypothesis of transmission line propagation was dropped. The equivalent plane waves in waveguide impinge the each resonator at different angles at different frequencies. Only the component of the magnetic field which is perpendicular to the particular resonator gives rise to the induced current. Therefore, the permeability has additional frequency dependence which inherently causes narrower frequency band. Strictly speaking, this structure cannot be considered as an isotropic effective medium. In order to investigate this effect, the additional 20 resonators were inserted into the waveguide and located perpendicularly to the central row (Fig. 2b). This caused broadening of the pass-band but did not change the general behavior. Finally, it was attempted to verify the existence of the backward wave propagation which has not been done in [8]. The second waveguide with the length of 70 mm was fabricated and filled with 16 resonators in the central row. Both waveguides were terminated with short circuit and input reflection coefficient was measured at three fixed frequencies inside the pass band. The results are shown in Fig. 4. It can be seen that at the fixed frequency, physically longer waveguide appears electrically shorter. It happened due to backward propagation which introduces positive phase shift of the signal (phase leads along the waveguide). Therefore, the waveguide behaves as it would behave the one-dimensional backward medium. From Fig.4, it can be also seen that the increase of the frequency causes the decrease of the phase of reflection coefficient. Thus, the backward material obeys Foster's theorem, as it was elaborated in [10].

4. CONCLUSIONS

It has been shown both theoretically and experimentally that rectangular waveguide filled with negative permittivity material supports propagation of backward waves below the cut-off frequency of dominant TE mode. The waveguide satisfies Foster's theorem, but the increase of the physical length of the waveguide is accompanied with the decrease of the electrical length.

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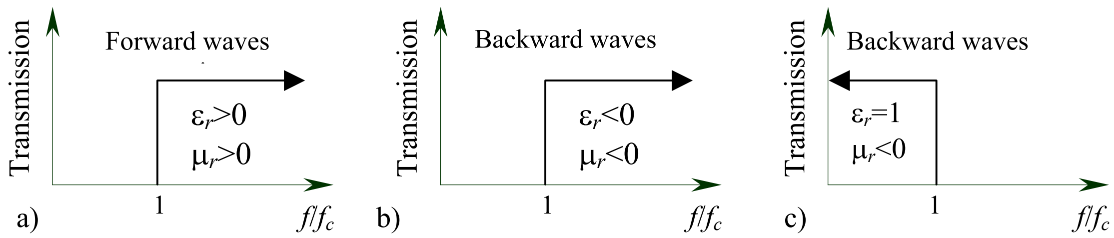

Fig. 1 The influence of filling material on the propagation in a waveguide
a) the case with $\varepsilon_r>0$, $\mu_r>0$ b) the case with $\varepsilon_r<0$, $\mu_r<0$ c) the case with $\varepsilon_r=1$, $\mu_r<0$

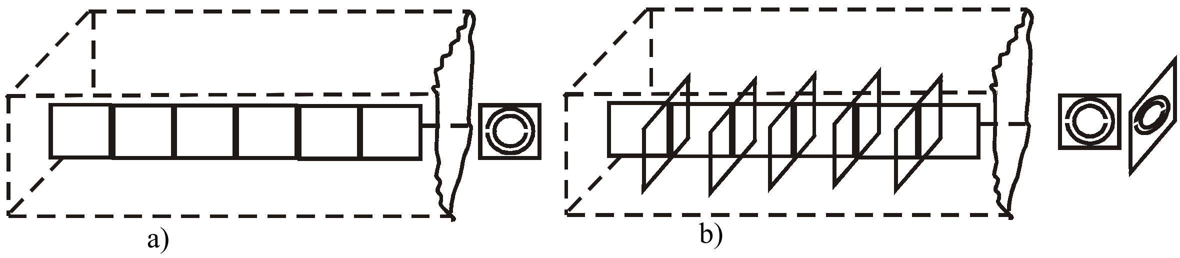

Fig.2 Experimental waveguide; split ring resonators stacked in one direction (a) and two directions (b)

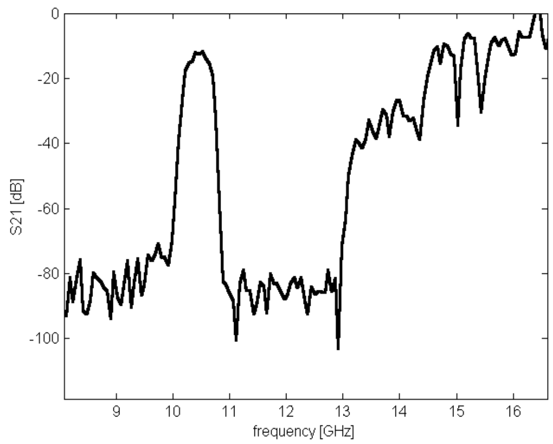

Fig. 3 Measured transmission coefficient of experimental waveguide

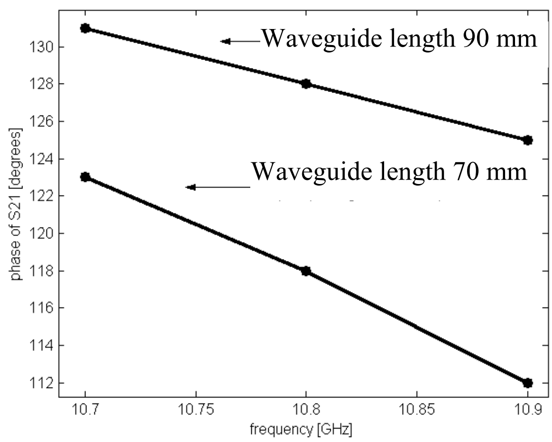

Fig. 4 Phase of measured reflection coefficient of the waveguide terminated with short circuit

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